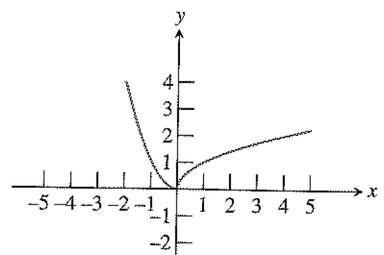


FIGURE 1.52 A piecewise-defined function. (Example 7)

EXAMPLE 7 Defining a Function Piecewise

Using basic functions from this section, construct a piecewise definition for the function whose graph is shown in Figure 1.52. Is your function continuous?

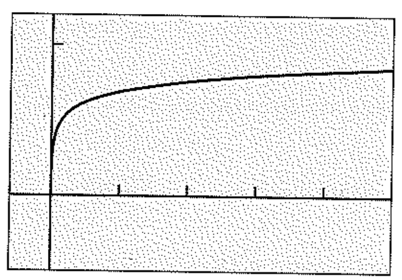
SOLUTION This appears to be the graph of $y = x^2$ to the left of $x = 0$ and the graph of $y = \sqrt{x}$ to the right of $x = 0$. We can therefore define it piecewise as

$$f(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$

The function is continuous.

Now try Exercise 47.

You can go a long way toward understanding a function's behavior by looking at its graph. We will continue that theme in the exercises and then revisit it throughout the book. However, you can't go *all* the way toward understanding a function by looking at its graph, as Example 8 shows.



[−600, 5000] by [−5, 12]

FIGURE 1.53 The graph of $y = \ln x$ still appears to have a horizontal asymptote, despite the much larger window than in Figure 1.42. (Example 8)

EXAMPLE 8 Looking for a Horizontal Asymptote

Does the graph of $y = \ln x$ (Figure 1.42) have a horizontal asymptote?

SOLUTION In Figure 1.42 it certainly *looks* like there is a horizontal asymptote that the graph is approaching from below. If we choose a much larger window (Figure 1.53), it still looks that way. In fact, we could zoom out on this function all day long and it would *always* look like it is approaching some horizontal asymptote—but it is not. We will show algebraically in Chapter 3 that the end behavior of this function is $\lim_{x\to\infty} \ln x = \infty$, so its graph must eventually rise above the level of any horizontal line. That rules out any horizontal asymptote, even though there is no *visual* evidence of that fact that we can see by looking at its graph.

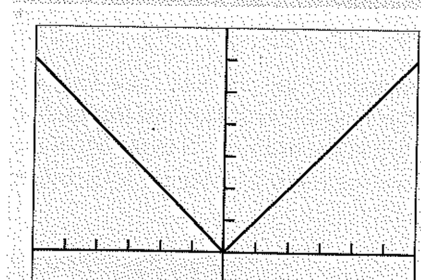
Now try Exercise 55.

EXAMPLE 9 Analyzing a Function

Give a complete analysis of the basic function $f(x) = |x|$.

SOLUTION

BASIC FUNCTION The Absolute Value Function



[−6, 6] by [−1, 7]

FIGURE 1.54 The graph of $f(x) = |x|$.

$f(x) = |x|$
Domain: All reals
Range: $[0, \infty)$
Continuous
Decreasing on $(\infty, 0]$; increasing on $[0, \infty)$
Symmetric with respect to the y-axis (an even function)
Bounded below
Local minimum at $(0, 0)$
No horizontal asymptotes
No vertical asymptotes
End behavior: $\lim_{x\to-\infty} |x| = \infty$ and $\lim_{x\to\infty} |x| = \infty$

Now try Exercise 67.

QUICK REVIEW 1.3 *(For help, go to Sections P.1, P.2, 3.1, and 3.3.)*

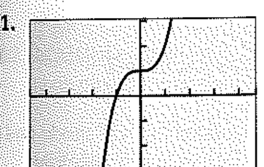
In Exercises 1–10, evaluate the expression without using a calculator.

1. $|-59.34|$
2. $|5 - \pi|$
3. $|\pi - 7|$
4. $\sqrt{(-3)^2}$
5. $\ln(1)$
6. e^0
7. $(\sqrt[3]{3})^3$
8. $\sqrt[3]{(-15)^3}$
9. $\sqrt[3]{-8^2}$
10. $|1 - \pi| - \pi$

SECTION 1.3 EXERCISES

In Exercises 1–12, each graph is a slight variation on the graph of one of the twelve basic functions described in this section. Match the graph to one of the twelve functions (a)–(l) and then support your answer by checking the graph on your calculator. (All graphs are shown in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.)

(a) $y = -\sin x$ **(b)** $y = \cos x + 1$ **(c)** $y = e^x - 2$
(d) $y = (x + 2)^3$ **(e)** $y = x^3 + 1$ **(f)** $y = (x - 1)^2$
(g) $y = |x| - 2$ **(h)** $y = -1/x$ **(i)** $y = -x$
(j) $y = -\sqrt{x}$ **(k)** $y = \text{int}(x + 1)$ **(l)** $y = 2 - 4/(1 + e^{-x})$

1.


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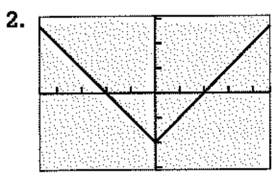

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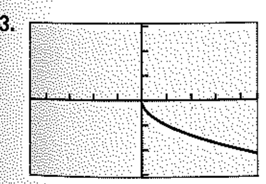

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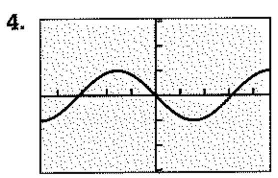

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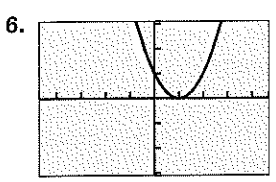

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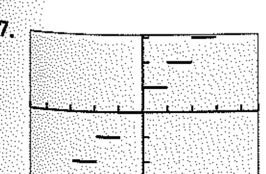

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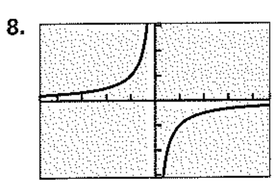

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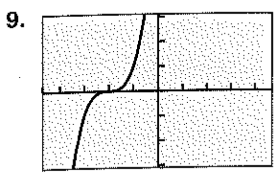

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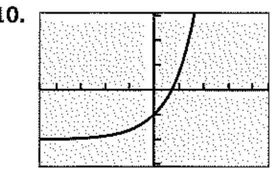

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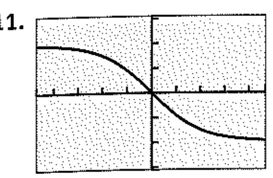
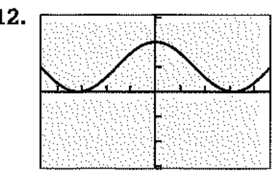


12.


In Exercises 13–18, identify which of Exercises 1–12 display functions that fit the description given.

13. The function whose domain excludes zero.

14. The function whose domain consists of all nonnegative real numbers.

15. The two functions that have at least one point of discontinuity.

16. The function that is not a *continuous function*.

17. The six functions that are bounded below.

18. The four functions that are bounded above.

In Exercises 19–28, identify which of the twelve basic functions fit the description given.

19. The four functions that are odd.

20. The six functions that are increasing on their entire domains.

21. The three functions that are decreasing on the interval $(-\infty, 0)$.

22. The three functions with infinitely many local extrema.

23. The three functions with no zeros.

24. The three functions with range {all real numbers}.

25. The four functions that do *not* have end behavior $\lim_{x \to +\infty} f(x) = +\infty$.

26. The three functions with end behavior $\lim_{x \to -\infty} f(x) = -\infty$.

27. The four functions whose graphs look the same when turned upside-down and flipped about the y-axis.

28. The two functions whose graphs are identical except for a horizontal shift.

In Exercises 29–34, use your graphing calculator to produce a graph of the function. Then determine the domain and range of the function by looking at its graph.

29. $f(x) = x^2 - 5$ **30.** $g(x) = |x - 4|$

31. $h(x) = \ln (x + 6)$ **32.** $k(x) = 1/x + 3$

33. $s(x) = \text{int } (x/2)$ **34.** $p(x) = (x + 3)^2$

In Exercises 35–42, graph the function. Then answer the following questions:

(a) On what interval, if any, is the function increasing? Decreasing?

(b) Is the function odd, even, or neither?

(c) Give the function's extrema, if any.

(d) How does the graph relate to a graph of one of the twelve basic functions?

35. $r(x) = \sqrt{x - 10}$ **36.** $f(x) = \sin (x) + 5$

37. $f(x) = 3/(1+e^{-x})$ **38.** $q(x) = e^x + 2$

39. $h(x) = |x| - 10$ **40.** $g(x) = 4 \cos (x)$

41. $s(x) = |x-2|$ **42.** $f(x) = 5 - \text{abs } (x)$

43. Find the horizontal asymptotes for the graph shown in Exercise 11.

44. Find the horizontal asymptotes for the graph of $f(x)$ in Exercise 37.

In Exercises 45–52, sketch the graph of the piecewise-defined function. (Try doing it without a calculator.) In each case, give any points of discontinuity.

45. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$ **46.** $g(x) = \begin{cases} x^3 & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$

47. $h(x) = \begin{cases} |x| & \text{if } x < 0 \\ \sin x & \text{if } x \ge 0 \end{cases}$ **48.** $w(x) = \begin{cases} 1/x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$

49. $f(x) = \begin{cases} \cos x & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$ **50.** $g(x) = \begin{cases} |x| & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

51. $f(x) = \begin{cases} -3 - x & \text{if } x \le 0 \\ 1 & \text{if } 0 < x < 1 \\ x^2 & \text{if } x \ge 1 \end{cases}$

52. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ |x| & \text{if } -1 \le x < 1 \\ \text{int } (x) & \text{if } x \ge 1 \end{cases}$

53. Writing to Learn The function $f(x) = \sqrt{x^2}$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain algebraically why the two functions are equal.

54. Uncovering Hidden Behavior The function $g(x) = \sqrt{x^2 + 0.0001} - 0.01$ is *not* one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it appears to be.

(b) Verify numerically that it is not the basic function that it appears to be.

55. Writing to Learn The function $f(x) = \ln (e^x)$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain how the equivalence of the two functions in (a) shows that the natural logarithm function is *not* bounded above (even though it *appears* to be bounded above in Figure 1.42).

56. Writing to Learn Let $f(x)$ be the function that gives the cost, in cents, to mail a letter that weighs x ounces. As of June 2002, the cost is 37 cents for a letter that weighs up to one ounce, plus 23 cents for each additional ounce or portion thereof.

(a) Sketch a graph of $f(x)$.

(b) How is this function similar to the greatest integer function? How is it different?

57. Analyzing a Function Set your calculator to DOT mode and graph the greatest integer function, $y = \text{int } (x)$, in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$. Then complete the following analysis.

BASIC FUNCTION
The Greatest Integer Function

$f(x) = \text{int } x$

Domain:
Range:
Continuity:
Increasing/decreasing behavior:
Symmetry:
Boundedness:
Local extrema:
Horizontal asymptotes:
Vertical asymptotes:
End behavior:

Standardized Test Questions

58. True or False The greatest integer function has an inverse function. Justify your answer.

59. True or False The logistic function has two horizontal asymptotes. Justify your answer.

In Exercises 60–63, you may use a graphing calculator to answer the question.

60. Multiple Choice Which function has range {all real numbers}?

(A) $f(x) = 4 + \ln x$
(B) $f(x) = 3 - 1/x$
(C) $f(x) = 5/(1 + e^{-x})$
(D) $f(x) = \text{int } (x - 2)$
(E) $f(x) = 4 \cos x$

61. Multiple Choice Which function is bounded both above and below?

(A) $f(x) = x^2 - 4$
(B) $f(x) = (x - 3)^3$
(C) $f(x) = 3e^x$
(D) $f(x) = 3 + 1/(1 + e^{-x})$
(E) $f(x) = 4 - |x|$

62. Multiple Choice Which of the following is the same as the restricted-domain function $f(x) = \text{int } (x)$, $0 \le x < 2$?

(A) $f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } x = 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

(B) $f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } 0 < x \le 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

(C) $f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x < 2 \end{cases}$

(D) $f(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 2 & \text{if } 1 \le x < 2 \end{cases}$

(E) $f(x) = \begin{cases} x & \text{if } 0 \le x < 1 \\ 1 + x & \text{if } 1 \le x < 2 \end{cases}$

63. Multiple Choice Increasing Functions Which function is increasing on the interval $(-\infty,\infty)$?

(A) $f(x) = \sqrt{3 + x}$
(B) $f(x) = \text{int } (x)$
(C) $f(x) = 2x^2$
(D) $f(x) = \sin x$
(E) $f(x) = 3/(1 + e^{-x})$

Explorations

64. Which is Bigger? For positive values of x, we wish to compare the values of the basic functions x^2, x, and $\sqrt{x}$.

(a) How would you order them from least to greatest?

(b) Graph the three functions in the viewing window $[0, 30]$ by $[0, 20]$. Does the graph confirm your response in (a)?

(c) Now graph the three functions in the viewing window $[0, 2]$ by $[0, 1.5]$.

(d) Write a careful response to the question in (a) that accounts for all positive values of x.

65. Odds and Evens There are four odd functions and three even functions in the gallery of twelve basic functions. After multiplying these functions together pairwise in different combinations and exploring the graphs of the products, make a conjecture about the symmetry of:

(a) a product of two odd functions.

(b) a product of two even functions.

(c) a product of an odd function and an even function.

66. Group Activity Assign to each student in the class the name of one of the twelve basic functions, but secretly so that no student knows the "name" of another. (The same function name could be given to several students, but all the functions should be used at least once.) Let each student make a one-sentence self-introduction to the class that reveals something personal "about who I am that really identifies me." The rest of the students then write down their guess as to the function's identity. Hints should be subtle and cleverly anthropomorphic. (For example, the absolute value function saying "I have a very sharp smile" is subtle and clever, while "I am absolutely valuable" is not very subtle at all.)

67. Pepperoni Pizzas For a statistics project, a student counted the number of pepperoni slices on pizzas of various sizes at a local pizzeria, compiling the following table:

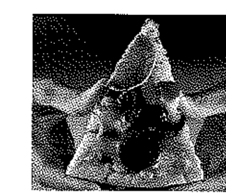


Table 1.10

Type of Pizza	Radius	Pepperoni count
Personal	4"	12
Medium	6"	27
Large	7"	37
Extra Large	8"	48

(a) Explain why the pepperoni count (P) ought to be proportional to the square of the radius (r).

(b) Assuming that $P = k \cdot r^2$, use the data pair (4, 12) to find the value of k.

(c) Does the algebraic model fit the rest of the data well?